References

[1] Strickfaden, W. B. and Geiler, K. L., "Probe Measurements of the Discharge in an Operating Electron Bombardment Engine," *AIAA Journal*, Vol. 1, No. 8, Aug. 1963, pp. 1815–1823.

[2] Knauer, W. and Poeschel, R. L., "Semi-annual Report—Discharge Chamber Studies for Mercury Bombardment Ion Thrusters," CR-72350, 1967, NASA.

[3] Knauer, W. et al., "Final Report—Discharge Chamber Studies for Mercury Bombardment Ion Thrusters," CR-72440, Sept. 1968, NASA.

[4] Masek, T. D., "Plasma Characteristics of the Electron Bombardment Ion Engine," TR 32-1271, April 1968, Jet Propulsion Lab., Pasadena, Calif.

[5] Mott-Smith, H. M. and Langmuir, I., "The Theory of Collectors in Gaseous Discharges," *Physical Review*, Vol. 28, Oct. 1926, pp. 727–763.

[6] Druyvesteyn, M., "Der Niedervoltbogen," *Zeitschrift für Physik*, Vol. 64, 1930, pp. 781–798.

[7] Medicus, G., "Simple Way to Obtain the Velocity Distribution of the Electrons in Gas Discharge Plasmas from Probe Curves," *Journal of Applied Physics*, Vol. 27, No. 10, Oct. 1956, pp. 1242–1248.

[8] Medicus, G., "Diffusion and Elastic Collision Losses of the 'Fast Electrons' in Plasmas," *Journal of Applied Physics*, Vol. 29, No. 6, June 1958, pp. 903–908.

Calculation of Relaxing Turbulent Boundary Layers Downstream of Tangential Slot Injection

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Nomenclature

C_f = skin-friction coefficient
l = mixing length
M_∞ = freestream Mach number
q = heating rate
s = slot height
S_t = Stanton number
T = temperature
t = slot lip thickness
U = longitudinal velocity
x,y = cartesian coordinates along and normal to the surface
$y_{s,1}$ = distance to inner boundary of mixing region
$y_{s,2}$ = distance to outer boundary of mixing region
y_c = slot half height
ψ = mixing angle (Fig. 1)
δ = total viscous layer thickness
λ = ratio of average specific mass flow in slot to value at edge of boundary layer

Subscripts

e = local external to boundary
max = maximum value
0 = reference value
t = isentropic stagnation conditions
w = wall

CURRENT interest exists in the possibility of using tangential slot injection for reducing heat transfer and skin friction on hypersonic vehicles.[1] Several low-speed experimental investigations of the downstream effects of tangential injection have been conducted which could provide information for the assessment of such a cooling system (see Ref. 2 for a partial list of these investigations). An accurate prediction method has been developed for the far field ($x/s > 30$) effects of tangential injection, primarily for the low-speed case.[3] Data also exist for tangential slot injection at supersonic and hypersonic speeds (Refs. 4, 5, and 6, for example). The present Note describes the application of a simple calculation method for compressible relaxing slot flows. Agreement with the data indicates that the simplified approach described here applies to the near slot ($x/s < 30$) as well as the far slot ($x/s > 30$) regions of the flow. This approach could be used, for example, in parametric studies to estimate the effects on the relaxation process of slot-to-edge total temperature ratio, slot height to boundary-layer height, and slot Mach number. The present method uses the turbulent boundary-layer computational procedure of Ref. 7 with the conservation equations for the mean flow solved by an implicit finite difference procedure and a turbulent Prandtl number relating the eddy viscosity and eddy conductivity. The solutions presented herein use a static turbulent Prandtl number[7] of 0.9. Note that the present method applies only to the matched static pressure case where the exit pressure of the slot flow essentially equals the value in the external flow, i.e., where mixing dominates the problem.

Before the method of Ref. 7 can be applied to relaxing tangential slot flows, appropriate modifications must be made to the mixing length model for eddy viscosity. Figure 1 illustrates the method employed here to model the mixing length for tangential slot flows. We assume the slot flow mixes with the initial boundary-layer flow in a region whose growth can be characterized to first order by a mixing angle ψ (Fig. 1a). This assumption allows computation of the nominal thickness of an effective mixing region as a function of distance downstream from the slot. With this model final results were fairly insensitive to the value of ψ assumed. On a test case to be considered in this Note, for example, increasing the ψ value from 4° to 6° decreased the cooling length (where T_{aw} first becomes larger than the slot flow total temperature) by 10%. Unpublished work by Beckwith and Bushnell at Langley Research Center indicates that the computation of the mixing of a trace species inserted in the initial boundary layer can be used to determine the mixing

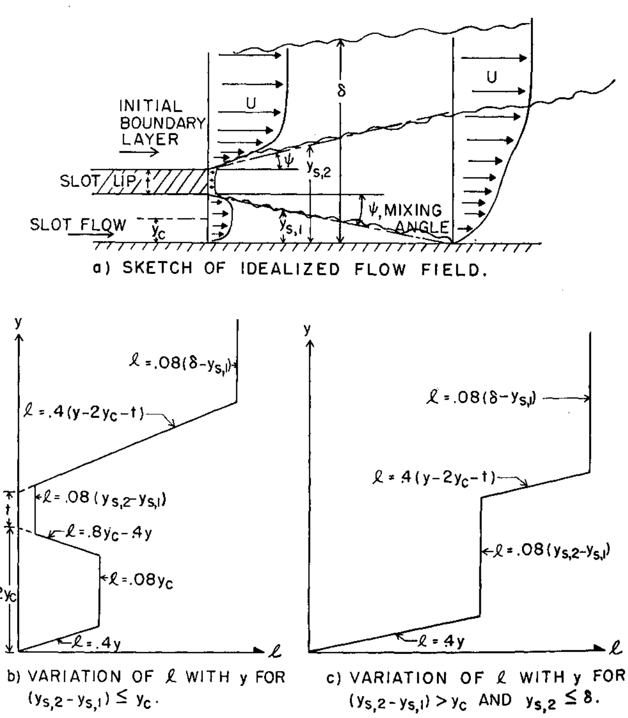

Fig. 1 Assumed mixing length distributions.

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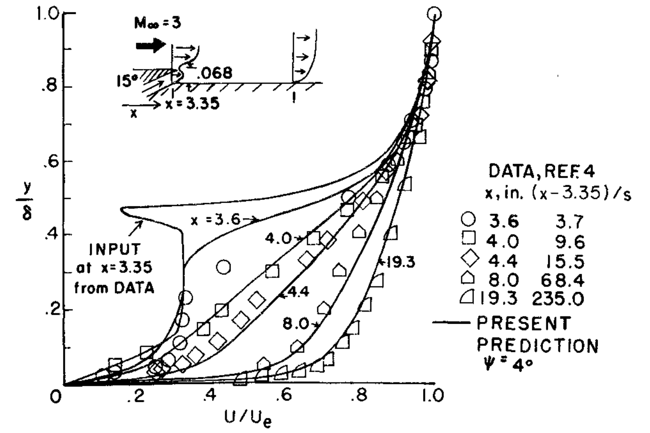

Fig. 2 Velocity profiles in relaxation region downstream of tangential slot injection, $\dot{m}_{slot} = 0.038$ lb/sec.

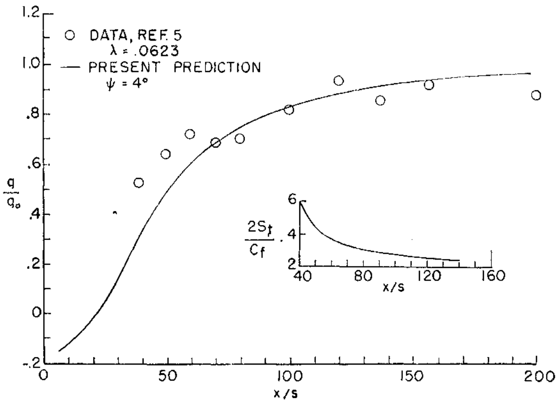

Fig. 3 Heat-transfer and Reynolds analogy distribution downstream of slot, $T_w/T_{t,e} = 0.66$, $q_o = 0.93$ Btu/ft² sec, $s = 0.010$-in., $M_\infty = 6.0$.

region boundaries. This method can replace the present ψ assumption to compute the relaxation process in slot flows, and removes some of the empiricism of the present method.

Mixing length variations used in the solution procedure are indicated on Fig. 1b and c. The present method assumes the maximum mixing length in the effective mixing region proportional to $y_{s,2} - y_{s,1}$ and a proportionality factor of 0.08 (an average value from the available data) to establish the maximum mixing length in the initial slot flow, the mixing region, and the outer boundary-layer flow. That is, $l_{max} = 0.08\delta_i$ where δ_i equals $y_c, y_{s,2} - y_{s,1}$ and $\delta - y_{s,1}$, respectively. The relaxation of the initial mixing length distribution is divided into three regions. Figure 1b indicates the l distribution for the first of these regions, where the width of the mixing region is less than the slot half height. The initial l distribution for the slot and initial boundary-layer flows (including the 0.4 Prandtl wall slope for the three wall regions) has superimposed upon it the maximum value for the mixing region, 0.08 $(y_{s,2} - y_{s,1})$. The method employs this maximum mixing region value $[0.08\ (y_{s,2} - y_{s,1})]$ wherever it exceeds the original l distribution. Figure 1c shows the assumed variation for the second stage of relaxation, where the mixing region becomes thicker than the slot half height, but the outer edge of the mixing region still lies inside the boundary layer $(y_{s,2} < \delta)$. Here the 0.4 slope applies near the wall until intersection with the maximum mixing region value. That is, the maximum mixing region level governs the variation in the inner portion of the boundary layer with the usual wall proximity effect (0.4 slope). The Van Driest damping function, as used in Ref. 7, modifies the l distribution for the near wall damping effects. The final relaxation stage $(y_{s,2} \geq \delta)$ uses the l distribution for developed turbulent boundary layers from Ref. 7.

Two comparisons will be shown between the results of the present method and experimental data. Cary shows a further comparison with a Mach 6 test case which indicates good agreement.[6] Figure 2 shows detailed profile data obtained at Mach 3 and computed results. The data were obtained downstream of a 0.068-in. slot with an initial total viscous layer thickness including the slot flow of 0.144 in. The prediction uses a ψ value of 4°. Considering the many assumptions involved in the method and the complexity of the slot-boundary-layer mixing problem, the indicated comparison of computed results with data is considered satisfactory. Figure 3 shows agreement of predicted and measured heating rates downstream of a small (0.010-in.) slot. These data[5] were obtained at Mach 6, a wall-to-total-temperature ratio of 0.66, and at approximately matched static pressure conditions with an initial viscous layer thickness the order of 0.8 in. By imposing the adiabatic wall boundary condition in the finite difference method, the adiabatic wall temperature distribution for this case was computed with the same input profiles used in the solution for $T_w/T_{t,e} = 0.66$. The combination of these T_{aw} values with the predicted q and C_f distribution for the $T_w/T_{t,e} = 0.66$ (constant) solution allows the determination of a Reynolds analogy factor $(2S_t/C_f)$ for this relaxing slot flow. This result appears in the insert on Fig. 3 and shows that the computed values are considerably above the conventional value of 1.0–1.3 for developed turbulent flows.[8] These higher values are presumably caused by the mixing of the cold gas with the hot external flow and the different relaxation behavior of the velocity and temperature profiles.

In conclusion, the present Note proposes a simple finite difference method, based on an assumed eddy viscosity model, for the solution of the relaxation region of tangential slot injection flows applicable to both the near slot and far slot regions. Computations using the method agree with experimental data at Mach numbers of 3 and 6. Typical results indicate that the Reynolds analogy factor in the relaxation downstream of tangential slot injection may be appreciably above the conventional values for developed turbulent boundary layers.

References

[1] Becker, J. V., "New Approaches to Hypersonic Aircraft," 7th Congress of the International Council of Aeronautical Sciences, Rome, Italy, Sept. 14–18, 1970.

[2] Burns, W. K. and Stollery, J. L., "The Influence of Foreign Gas Injection and Slot Geometry on Film Cooling Effectiveness," *International Journal of Heat and Mass Transfer*, Vol. 12, 1969, pp. 935–951.

[3] Kacker, C. C., Pai, B. R., and Whitelaw, J. F., "The Prediction of Wall Jet Flows With Particular Reference to Film Cooling," *Progress in Heat and Mass Transfer*, Vol. 2, Pergammon Press, New York, 1969.

[4] Peterson, J. B., Jr. et al., "Further Investigation of Effect of Air Injection Through Slots and Porous Surfaces on Flat Plate Turbulent Skin Friction," TN 3311, March 1966, NASA.

[5] Parthasarathy, K. and Zakkay, V., "Turbulent Slot-Injection Studies at Mach 6," ARL 69-0066, April 1969, Aerospace Research Labs., Wright-Patterson Air Force Base, Ohio.

[6] Cary, A. M., Jr. and Hefner, J. N., "Film Cooling Effectiveness in Hypersonic Turbulent Flow," *AIAA Journal*. Vol. 8, No. 11, Nov. 1970, pp. 2090–2091.

[7] Bushnell, D. M. and Beckwith, I. E., "Calculation of Nonequilibrium Hypersonic Turbulent Boundary Layers and Comparisons with Experimental Data," *AIAA Journal*, Vol. 8, No. 8, Aug. 1970, pp. 1462–1469.

[8] Cary, A. M., Jr., "Summary of Available-Information on Reynolds Analogy for Zero-Pressure-Gradient, Compressible, Turbulent-Boundary-Layer Flow," TN D 5560, Jan. 1970, NASA.